

Structural Representation of Measurement Error Bias

Bias	Causal Diagram
1 Uncorrelated errors under sharp null: no treatment effect: Under sharp null, assuming confounders are not measured with error, uncorrelated measurement errors are generally not expected to lead to bias.	
2 Uncorrelated errors under treatment effect: Outside sharp null, uncorrelated measurement errors distort targeted effects.	
3 Correlated errors: Related, systematic errors in A and Y measurements that are related.	
4 Directed error: exposure effects error of outcome: A affects Y's measurement error.	
5 Directed error: outcome affects error of exposure: Y affects A's measurement error.	
6 Correlated/directed error: Both systematic and correlated errors in A and Y measurements are from an unmeasured source of dependency.	

Key: A denotes the treatment; Y denotes the outcome; U denotes an unmeasured confounder; L denotes measured confounders; \longrightarrow asserts causality; $(X) \longrightarrow X'$ indicates a latent variable X measured by proxy X' ; \longrightarrow indicates a path for bias linking A to Y absent causation; $- \longrightarrow$ biased path for treatment effect in the target population; (X) indicates that conditioning on X introduces bias; $U_X \text{---} \circ \text{---} X'$ indicates that the error in a measured variable X' modifies the effect of $A \rightarrow Y$, such that the $ATE_{\text{target}} \neq \widehat{ATE}$