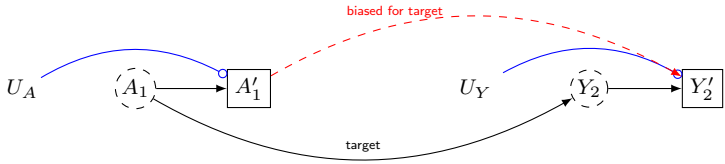
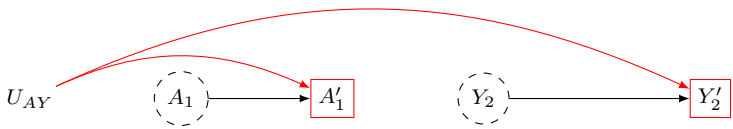
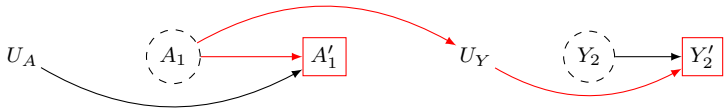
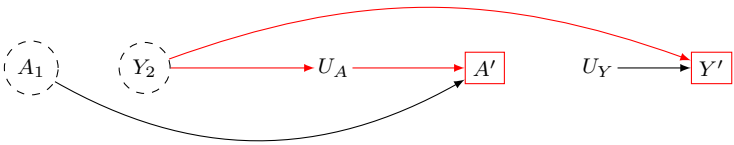
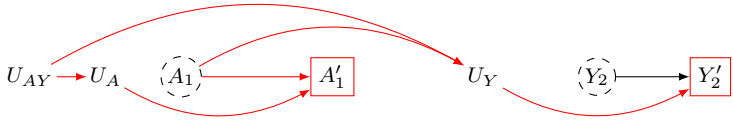
Structural Representation of Measurement Error Bias

	Bias	Causal Diagram
1	**Uncorrelated errors under sharp null: no treatment effect**: Under sharp null, assuming confounders are not measured with error, uncorrelated measurement errors are generally not expected to lead to bias.	
2	**Uncorrelated errors under treatment effect**: Outside sharp null, uncorrelated measurement errors distort targeted effects.	
3	**Correlated errors**: Related, systematic errors in A and Y measurements that are related.	
4	**Directed error: exposure effects error of outcome**: A affects Y's measurement error.	
5	**Directed error: outcome affects error of exposure**: Y affects A's measurement error.	
6	**Correlated/directed error**: Both systematic and correlated errors in A and Y measurements are from an unmeasured source of dependency.	

Key: A denotes the treatment; Y denotes the outcome; U denotes an unmeasured confounder; L denotes measured confounders; $\longrightarrow$ asserts causality; $\boxed{X}\longrightarrow\boxed{X'}$ indicates a latent variable X measured by proxy X'; $\longrightarrow$ indicates a path for bias linking A to Y absent causation; $\dashrightarrow$ biased path for treatment effect in the target population; $\boxed{X}$ indicates that conditioning on X introduces bias; $U_X \multimap X'$ indicates that the error in a measured variable X' modifies the effect of $A \to Y$, such that the $ATE_{\text{target}} \neq \widehat{ATE}$